THE FOURTH REGULAR MEETING OF THE SOUTHWESTERN SECTION.

THE fourth regular meeting of the Southwestern Section of the Society was held at the University of Nebraska, Lincoln, Nebraska, on Saturday, November 26, 1910. The following members of the Society were present:

Professor W. C. Brenke, Professor E. W. Davis, Professor E. P. R. Duval, Professor C. C. Engberg, Professor E. R. Hedrick, Professor O. D. Kellogg, Miss H. H. MacGregor, Professor W. H. Roever, Professor O. Schmiedel, Professor J. W. Young.

The morning session was opened at 10 A. M. and the afternoon session at 2 P. M., Professor Davis presiding. St. Louis was fixed as the next place of meeting of the section, and the following program committee elected: Professor Roever (chairman), Professor Brenke, Professor Kellogg (secretary). On Friday evening a smoker was held at the home of Professor Davis, at which some of the problems of the mathematical departments of the state universities were informally discussed, and the attending members lunched together on Saturday.

The following papers were presented at this meeting:

(1) Professor G. A. MILLER: "Abstract definition of all the substitution groups whose degrees do not exceed seven."

(2) Dr. ELIZABETH R. BENNETT: "Primitive groups of degree 20."

(3) Professor W. C. BRENKE: "On the series of zonal harmonics."

(4) Professor A. B. FRIZELL: "On certain transfinite permutations."

(5) Professor A. D. PITCHER: "Properties of certain classes of sequences."

(6) Professor E. W. DAVIS: "Imaginaries on a cubic" (preliminary communication).

(7) Dr. LOUIS INGOLD: "Curves in a function space."

(8) Dr. LOUIS INGOLD: "Surfaces in a function space."

(9) Professor FLORIAN CAJORI: "Horner's method of approximation anticipated by Ruffini."

(10) Professor W. H. ROEVER: "Southerly deviation of falling bodies. Second paper."

(11) Professor J. W. YOUNG: "On fundamental regions for cyclical groups of linear transformations on two complex variables."

(12) Professor O. D. KELLOGG: "Green's integral for multiply connected regions."

(13) Professor E. R. HEDRICK: "On assemblages with closed derivatives."

Dr. Bennett was introduced by Professor Davis. The papers of Professors Miller, Frizell, and Pitcher were read by title. Dr. Ingold's papers were presented by Professors Kellogg and Hedrick, respectively, and Professor Cajori's paper by Professor Davis. Abstracts of the papers follow below.

1. In addition to giving a list of possible abstract definitions of the substitution groups whose degrees do not exceed seven, and an explanation of these and other possible abstract definitions, Professor Miller proves several general theorems relating to such definitions. Among these are the following: If n operators of order two are such that the product of any three of them is also of order two, they generate a group which involves an abelian subgroup composed of half the operators of the group, and each of the remaining operators is of order two and transforms each operator of this abelian subgroup into its inverse. Moreover, if this abelian group involves no more than $n-1$ invariants, the entire group of twice its order can always be generated by n such operators of order two. The necessary and sufficient conditions that the augmented right co-sets

$$H + HS_2 + HS_3 + \cdots + HS_\rho$$

constitute a group are that they include the product of any two of the operators $S_2, S_3, \cdots, S_\rho$ and that they also include the inverse of each operator in these co-sets. New abstract definitions for the general symmetric and alternating group of degree n were also given.

2. Dr. Bennett proves in this paper certain theorems concerning the transitive constituents of the maximal subgroup G_1 of a primitive group G. The primitive groups of degree 20 are then determined and it is shown that only four such groups exist. These are the symmetric and alternating groups of degree 20 and the two well-known Mathieu groups of degree $p+1$ and orders $p(p^2-1)$ and $\frac{1}{2}p(p^2-1)$.

3. In Professor Brenke's paper the following theorem is obtained concerning the series

$$a_1X_1 + a_2X_2 + \cdots + a_nX_n + \cdots,$$

where X_n denotes the Legendrian polynomial of degree n in x:

The above series converges uniformly in any interval (c_1c_2) lying between -1 and $+1$ and not including either of those points, provided that

(1) it converges at one point of (c_1c_2);

(2) the series $\sum_1^\infty A_nX_n$, where

$$A_n = \frac{n(n-1)}{2n-1}a_{n-1} - \frac{(n+1)(n+2)}{2n+3}a_{n+1},$$

converges uniformly in (c_1c_2);

(3) $$\lim_{n=\infty} a_n/\sqrt{n} = 0.$$

If the sum of the series be denoted by $f(x)$, this function will have in (c_1c_2) a continuous first derivative given by the equation

$$f'(x) = \frac{2a_1}{3(1-x^2)} + \frac{1}{x^2-1}\sum_1^\infty A_nX_n.$$

Example: Applied to the series

$$\frac{3}{2}X_1 + \frac{7}{3.4}X_3 + \frac{11}{5.6}X_5 + \cdots,$$

this gives

$$f'(x) = \frac{1}{1-x^2}, \quad \text{hence} \quad f(x) = \frac{1}{2}\log\frac{1+x}{1-x}.$$

4. In this paper Professor Frizell develops a procedure for producing sets of permutations of the elements of an ω series which exhibit all ordinal types of the second class.

The prime numbers form an ω series, the products of two, three, $\cdots$, n primes form series of types ω^2, ω^3, $\cdots$, ω^n respectively. Thus the set of all products of primes (i. e., the series

of natural numbers) may be arranged in type ω^ω. Repetitions of this process combined with that of finite permutation explained in a previous paper yield permutations of the set of all natural numbers arranged in a series of type Ω.

The paper will appear in the *Science Bulletin* of the University of Kansas.

5. The General Analysis of Moore relates to certain properties of classes of real valued functions of a general variable. In this paper Professor Pitcher discusses certain classes of sequences (functions the range of whose variable is the denumerable class $1, 2, \cdots, n, \cdots$), showing which of the above mentioned properties are possessed by each class. The classes discussed are (*a*) all convergent series, (*b*) all convergent sequences, (*c*) all ultimately constant sequences. Each of these classes possesses properties which make the methods and theorems of general analysis applicable to it. The so-called $*$ extension of each of the above classes is computed.

6. Professor Davis considers the intersections with the cubic of a pencil of lines passing through a point upon the cubic. The imaginary intersections are represented by vectors along lines of the pencil joining points on a certain auxiliary cubic to points on a sextic complementary (in Poncelet's sense) to the original cubic. Through every point in the plane pass three such auxiliary cubics.

7. In Dr. Ingold's first paper use is made of the well known analogy between ordinary vectors and functions of a real variable x on an interval, say $a \leqq x \leqq b$; functions $f(x; s)$ are considered which involve a parameter s besides the variable x; the function f is regarded as the analogue of the defining vector of a curve in ordinary space. The extension of Frenet's formulas is used to define a set of functions analogous to the normal vectors of a curve. The usual vector formulas for curves in an n-dimensional space are then reproduced in terms of the function f and its derivatives with respect to the parameter s.

8. The second paper of Dr. Ingold contains a discussion of functions involving several parameters $u_1, u_2, \cdots, u_n$ besides the variable x on the interval $a \leqq x \leqq b$. A function

$f(x; u_1, u_2, \cdots, u_n)$ is regarded as the analogue of the defining vector of an n-dimensional surface. Functions analogous to normals to a surface are defined in terms of the first and second partial derivatives of f with respect to the parameters $u_1, \cdots, u_n$. The investigation is then limited to the case in which, for given values of the parameters u, the normals are all numerical multiples of the same function N and in which the derivatives of N with respect to the parameters are all linearly expressible in terms of the derivatives of f. With these restrictions on f the principal formulas of the differential geometry of n-dimensional surfaces in a $(n+1)$-dimensional (euclidean) space, are obtained in terms of f and its derivatives.

9. Professor Cajori shows that Paolo Ruffini published in 1804 at Modena in Italy a monograph which describes a method of approximation to the real roots of numerical equations which is almost identical with the method given by W. G. Horner in 1819 in England. The difference is unimportant: the coefficients of the transformed equation appear, with Ruffini in a column, with Horner along a diagonal line, slanting upward from left to right.

10. In computing the southerly deviation of a falling body, one can not, with impunity, neglect the curvature of the lines of force in the neighborhood of the point P_0 (fixed with respect to the rotating earth) from which the body falls. Unless the lines of force of the rotating field are rectilinear (which, in particular, is the case when the field is constant), a plumb-line P_0R supported at P_0 does not coincide with the tangent t at P_0 to the line of force of the rotating field which passes through P_0. Some writers, Gauss for instance, measure the southerly deviation from the cone of revolution generated by t. In experiments, on the other hand, the southerly deviation is measured on the equipotential surface (of the rotating field) which passes through the plumb-bob R, from the circle of latitude generated by R. Therefore, in order to compare the results of experiment with those of theory, the latter method must be used. This method is the same as that described by Professor Roever in an earlier paper.*

In his present paper, a formula for the southerly devi-

* BULLETIN, vol. 16 (1910) p. 228.

ation (hereafter denoted by S. D.) is expressed in terms of the potential function $U_1 = f_1(r, z)$ (where r and z denote the distances of a general point from the axis of rotation OZ and the plane of the equator of the earth) of the earth's fixed field of force. The formula is of the form

$$\text{S.D.} = Ah^2 + \cdots, \tag{1}$$

where $h = P_0R$ is the distance through which the body falls and A is a function of the earth's angular velocity of rotation ω and the first and second derivatives of f_1 for the point P_0, (r_0, z_0).

If, in particular, we assume the (rotating) field of force in the neighborhood of P_0 to be constant and of intensity g,

$$f_1 = f_2 - \tfrac{1}{2}\omega^2 r, \quad \text{where} \quad f_2 = -g\cos\phi\cdot r - g\sin\phi\cdot z,$$

in which the constant ϕ is the astronomical latitude at P_0. For this assumption formula (1) becomes

$$\text{S.D.} = \tfrac{3}{8}\sin\phi\cos\phi\frac{\omega^2}{g}h^2, \tag{2}$$

or

$$\tfrac{1}{8}\sin\phi\cos\phi\, g\omega^2 t^4, \quad \text{since} \quad h = \tfrac{1}{2}gt^2 + \cdots$$

This is the formula given by Gauss* and later writers.

If, on the other hand,

$$f_1 = \frac{M}{\rho}\left[1 + \frac{\epsilon}{3}\frac{\rho_1^2}{\rho^2}\frac{r^2 - 2z^2}{\rho^2}\right],\dagger$$

where M is the mass of the earth, $\rho = \sqrt{r^2 + z^2}$, ρ_1 the mean radius of the earth and ϵ the constant .00167, formula (1) takes the form

$$\text{S.D.} = .0055\sin 2\phi_0\cdot\frac{h^2}{\rho_0}, \tag{3}$$

where ϕ_0 is the geocentric latitude of P_0 and ρ_0 is ρ for P_0.

* Gauss, Werke, vol. 5, 1867, p. 502.

† A formula given by H. Poincaré, Figures d'équilibre d'une masse fluide (1902), p. 107.

This formula gives values which are four and one half times as great as those of formula (2).

11. The purpose of Professor Young's note is to call attention to a very simple method of obtaining fundamental regions for cyclical groups of linear fractional transformations on two complex variables. The simplicity of the method is due to the fact that the determination of a fundamental region for a group of the specified kind is made to depend on the construction of a fundamental region for a simply isomorphic group on a single complex variable. The method, moreover, may be readily extended to the case in which the number of variables is n and to certain restricted types of groups which are not cyclical and not linear.

12. Continuing the work which appeared in the *Transactions* of January, 1908, Professor Kellogg makes an immediate extension to multiply connected regions. By means of the results Green's integral formula

$$u(x, y) = \frac{1}{2\pi} \int u(s) \frac{\partial G}{\partial n} ds$$

is rigorously established without any presuppositions concerning $u(x, y)$. From this criteria are obtained for the unique determination of harmonic functions by their boundary values in the case that these are not continuous.

13. At the last meeting of the Section, Professor Hedrick proved that a few important theorems of the theory of point sets hold true for any compact set of objects for which the first derived set is closed. In the present paper the hypothesis is added that any element of the set is enclosable in the interior of a family of diminishing assemblages in a uniform manner. With this hypothesis practically all of the fundamental theorems of point sets are shown to hold for general sets of objects; the list of theorems includes the general Heine-Borel theorem, the Cantor-Bendixon theorem, the Cauchy fundamental theorem, the oscillation theorem, and many others. No distance notion is used; those theorems which usually involve distance are reworded to avoid that concept.

O. D. KELLOGG,
Secretary of the Section.

UNIVERSITY COURSES IN MATHEMATICS AND THE MASTER'S DEGREE.

*PROVISIONAL REPORT OF THE AMERICAN SUBCOMMITTEE OF THE INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS.**

I. *Preliminary Remarks.*

In preparing this report the committee has found the greatest difficulty in securing reliable information on the various topics considered. Nothing of the kind has been attempted since Cajori's report of 1890, "On the Teaching and History of Mathematics in the United States." † Valuable as this is, it covers only the period of the beginning of graduate study; and except for two brief statistical notes in the Reports of the U. S. Commissioners of Education and scattered items of information in the BULLETIN of the American Mathematical Society, there exists no published information on the development of graduate instruction in mathematics since 1890. The Committee has, therefore, been compelled to gather its material almost entirely at first hand, relying mainly on personal correspondence and on statements contained in catalogues and other publications of American universities. It is needless to say that only a limited amount of information can be obtained in the former way, while as for the latter, it is too frequently the case that the facts desired are either very hard to get at, or are not given at all.

Over thirty colleges and universities in the United States offer graduate work in mathematics, but less than fifteen have given a doctor's degree in that subject within the last five years. The following twenty-four offer courses of an advanced character and report three or more graduate students for the year

* *International Commission on the Teaching of Mathematics.* Committee XIV. — Graduate Work in Universities. Subcommittee 1 — Courses of Instruction and the Master's Degree. Chairman, D. R. CURTISS, Northwestern University; EDWARD KASNER, Columbia University; A. C. LUNN, University of Chicago. This report was submitted to the American Commissioners in November, 1910. The report of Subcommittee 3 was published in the November, 1910, BULLETIN.

† U. S. Bureau of Education, Circular of Information No. 3, 1890.

1908–09: Bryn Mawr, California, Chicago, Cincinnati, Clark, Colorado, Columbia, Cornell, Harvard, Illinois, Indiana, Iowa, Johns Hopkins, Michigan, Missouri, Nebraska, Northwestern, Pennsylvania, Princeton, Leland Stanford, Syracuse, Virginia, Wisconsin, Yale. To this somewhat arbitrary list the greater part of the statistics of this report refer, but even in the field thus limited it has been impossible to obtain entirely reliable information on all the points covered. Accordingly, though the numerical statements here given are believed to be in the main correct, it has seemed best not to give detailed statistics for any particular institution.

II. *Historical Sketch.*

Genuine graduate work in mathematics may be said to date from the founding of Johns Hopkins University, and the assumption of his duties there by Sylvester, in the year 1876–1877. It is true that courses of an advanced character had previously been given in various institutions, notably at Harvard, where Benjamin Peirce taught for nearly fifty years and exerted a profound influence on American mathematical development; but this earlier work was never of an exclusively graduate character, and was carried on with frequent interruptions due to the lack of properly prepared students. Johns Hopkins was founded primarily as a graduate school and that department has always overshadowed the college. Its great success proved a powerful stimulus to advanced work along all lines in this country and led ultimately to the founding of separate graduate departments in most of the leading American universities.

The group of eight or ten whom Sylvester soon gathered about him, and their successors, carried something of their master's enthusiasm with them when they went out to other institutions, and in the course of a few years advanced instruction in mathematics was begun in a number of other colleges and universities. Almost as important were some of the indirect results of this influence; for although the development at Harvard was independent of the Johns Hopkins movement, it is significant that this was also the period when graduate work in mathematics was placed on a permanent basis at Harvard.

The course of lectures delivered at Johns Hopkins by Cayley in 1882 had fourteen regular attendants, and added materially

to the force of this movement. After Sylvester's departure in 1883 his pupils and associates continued his influence. Cajori reports nearly a dozen institutions in which graduate instruction was offered in the decade 1880–1890. Among them he mentions specifically Harvard, Cornell, Princeton, Virginia, South Carolina, Texas, Michigan, and Wisconsin, as well as Johns Hopkins, and indirectly refers to the work at Yale.

After the departure of Sylvester and Cayley it was natural that the more ambitious students should wish to continue their work under other mathematicians of the same order. Hence we find that by 1885 a number of Americans were studying in Europe, mainly in the German universities. It was no new thing for an American to receive a doctor's degree abroad, but up to this time such cases had been relatively few. From 1885 to 1893, however, American mathematical study continued more and more to derive its inspiration from the seminars of Leipzig, Berlin, and Göttingen. The period of English influence was replaced by one in which a German training was the greatest desideratum, and if a single man is to be designated as Sylvester's successor, it is probably correct to say that the majority of Americans who studied mathematics in European universities received their training at this time in the seminar of Klein. This period was marked by the institution of strong mathematical departments in the newly founded Clark University (1889) and the University of Chicago (1892) and culminated in the Mathematical Congress held at Chicago in 1893 during the World's Fair, and Klein's Evanston lectures.

By 1893 Americans trained abroad had become an influential, and later a controlling element, and from that time American mathematics stood on its own feet. At present doctor's degrees in mathematics obtained each year at home and abroad are about in the ratio of five to one. It is, however, true that many still go to the great universities of Europe, and it will be shown in a later section of this report that at least half of those giving graduate instruction in this country have studied abroad.

III. *Organization and Personnel.*

At present practically all of the leading American universities have organized their graduate work into a separate school or college. With two or three exceptions, however, there is no division of the teaching force into graduate and undergraduate

faculties. In particular, the same department of mathematics gives both advanced and elementary instruction, though not all its members may participate in the former. The number of those who give no undergraduate courses is very small. The work of the department is usually directed by one of the professors, designated as the head or chairman.

An examination of publications of the twenty-four institutions mentioned in Section I shows that the total mathematical teaching force, not including student assistants, was 202 in the year 1908–1909. Of these 136 offered advanced courses, but probably a considerably smaller number actually gave such work, since it will appear in Section IV that the total number of graduate students specializing in mathematics at these institutions was less than 250. It thus seems that only about half the total teaching force at these institutions had a part in graduate instruction. It should be added, however, that in a number of cases large technological departments necessitate a great amount of elementary mathematical work and increase the number of instructors who must devote all their time to such courses. In only two or three institutions was advanced work given by every member of the department.

A further inquiry as to the amount of advanced work given by the 136 instructors mentioned in the preceding paragraph shows that 45 undertook courses announced as primarily for graduates amounting to more than three hours per week. In only five universities were there four or more men offering more graduate work than this, while in ten there was either only one, or else no member of the department who offered more. It thus appears that the greater part of advanced instruction is in the hands of about one fifth of the total teaching force. Naturally these are the older men, as a rule, but it is also evident that a considerable number of others give a limited amount of graduate work. In many institutions each of the younger instructors offers a single graduate course.

In the past the complaint has frequently been heard that American professors are overburdened. That this is still the case, even with men giving a considerable amount of advanced work, is evident from the fact that 9 of the 45 just mentioned have a teaching schedule of from 14 to 16 hours a week. However, there has undoubtedly been an improvement of late in this respect, and the average for all the 45 is about 11 hours per week, a figure that still leaves much to be desired.

As to the preparation of those announcing graduate courses, it appears that nearly five-sixths have obtained the doctor's degree, and probably half of the remainder have had an equivalent training. The instructional force would thus seem to be well equipped. More than half have received training in European universities; a partial list of those who have spent at least one year abroad shows 50 who have studied in Germany, 14 in England, 5 in France, and 1 in Italy.

There is a marked tendency in some of the older universities to recruit their teaching staff from their own students. Even when a period of residence elsewhere has intervened such a practice has some disadvantages, but it also tends to give a permanent character to the work of the department.

IV. *The Graduate Students.*

It may be said generally that the one prerequisite for status as a graduate student in an American university is the possession of a bachelor's degree. In many cases it is announced that this degree must have been given by an institution of equal rank, but as a matter of fact this stipulation is almost never enforced. The result of this is that there is no uniformity of preparation among those classed as graduate students, and frequently such men are taking work as elementary as the first course in the calculus. There are usually, however, more definite requirements to be fulfilled for admission to candidacy for a master's or a doctor's degree, but some institutions make no distinction of this sort and none require all graduate students to qualify as candidates for those degrees. In view of the wide range of attainment which presents itself among those classed as graduate students, statistics on this subject have only a relative value.

Another difficulty that arises is in the estimate of the work of the summer schools which have sprung up in the last fifteen years in connection with most of the larger institutions. Many of these have an attendance of from 500 to 1,500, a considerable proportion being teachers in secondary schools. The instruction is given by regular members of the university, or others specially engaged. The brief duration of these sessions, usually six weeks, and the fact that this is vacation work, very often brings about a serious lowering of standards. Usually, only a small amount of advanced work is offered, and the pub-

lished lists of students give no clue as to the number taking mathematical courses.* For this reason we shall here attempt to give no statistics as to graduate work in summer schools beyond the statement that ten institutions out of the twenty-four appear to give such courses, and that at least in two or three western institutions the enrollment in these classes is larger than the registration in regular advanced work.

The total number of graduate students in attendance on mathematical courses for more than one term or quarter of the year 1908–09 was probably about 400. The 24 institutions especially investigated report 295, but many indicate only the department in which major work is carried, so that a considerable number specializing in physics and astronomy are not included in this figure. It is easier to obtain a fairly reliable estimate of how many have mathematics as their primary subject. The institutions mentioned report 225, not including summer students and those enrolled for only one term or quarter. Possibly 25 more should be added for universities outside this list, making a total of 250, a number less than twice that of instructors offering advanced courses. It may be interesting to note that 40 of the above-mentioned 225 were women. Only two of the leading universities now restrict their graduate attendance to men.

In this connection it would be highly desirable to present some statistics on the growth of graduate work in mathematics and its relation to the growth of graduate study in general. Unfortunately, little is to be found bearing on this point. In the report of the United States Commissioner of Education for the year 1896–1897 (volume 2, page 1649) the number of mathematical students in 24 institutions is given as 247 out of a total graduate enrollment of 3,204, but this probably includes summer registration and students specializing in other subjects. Since that time the number of graduate students in all departments has more than doubled, and mathematics would seem to have nearly kept pace with the general development. In 1904 the same authority (volume 2, page 1424) gives figures for ten

* The summer quarter at the University of Chicago differs considerably from the summer session in other institutions in that it is of equal duration with each of the other three quarters, and is in other respects the same except in the character of the attendance. It is, however, divided into halves, and many students attend for six weeks only. It may be noted that the number of graduate students in mathematics enrolled in this one session is three times the average registration for any other quarter.

institutions, showing 184 graduate students specializing in mathematics, inclusive of about 80 in summer sessions. This total has been increased by nearly 40 per cent in the last five years. It appears, however, that most of this recent growth has been confined to three or four institutions. Columbia reports an increase of over 300 per cent, and there has also been noteworthy development at the University of Illinois, large sums having been appropriated for the graduate school within the last two or three years.

Another indication of the increase of graduate students is afforded by a statement in the BULLETIN of the American Mathematical Society (volume 16, number 2, page 95) in which the number receiving the doctorate in mathematics from American universities in each of the years 1899–1909 is given as, — 13, 11, 18, 8, 7, 14, 21, 11, 13, 22, 15. It may be noted that the average for the first three is nearly six-sevenths that of the last three. Since regulations affecting the doctor's degree are not generally more stringent than they were ten years ago, and no larger percentage of Americans are now studying abroad, these figures naturally lead one to inquire whether advanced work is really increasing as fast as the enrollment would indicate. It is possible that special courses for teachers have largely swelled the totals without materially adding to the amount of research. Another cause for the small and apparently decreasing percentage of doctorates may be the great demand for teachers of mathematics, due largely to the recent growth of technological work; this pressure has probably led many to discontinue their studies earlier than they would otherwise have done. Some light is shed on this point by the fact that in the year 1908–1909 about sixty took the master's degree with mathematics as a major subject. Probably most of these had only that degree in view and terminated their graduate work as soon as it was obtained.

The bulk of advanced work is done by students receiving financial aid, or on salary as subordinate members of the teaching staff. To be more specific, out of the total of 225 in the institutions mentioned, 57 received fellowships and scholarships, paying from $100 to $800 per year, and 61 were assistants or instructors. We may safely assume that a considerable proportion of the remaining 107 were secondary teachers or others taking but one course. There would thus seem to be relatively few giving the major part of their time to study without remu-

neration, and in fact only four institutions of the twenty-four have as many as six students of this class. The large number of assistants and instructors in graduate classes gives a hint of what is often a most unfortunate state of affairs. These are men who should be devoting all their time to graduate study, but they are usually so loaded down with class-work as to be unable to do justice to the courses they attempt to carry, and so poorly remunerated that they cannot save enough to provide for the additional year or two of research work necessary for the doctorate.

An examination of the migration of graduate students of mathematics develops some very interesting facts. In the institutions mentioned, 70 per cent received their bachelor's degree at another college or university, and about 50 per cent had not received their undergraduate training at any of the other places on that list. One may gather that there is no great disposition among students in universities maintaining advanced work to go elsewhere for further study after receiving the bachelor's degree, but that there is a large flow from the small colleges to the graduate schools. This movement is a source both of strength and weakness to the latter; of strength from the constant accession of fresh and in the main good material; of weakness in that the preparation of many of these men is very defective. Only 25 per cent of the total enrollment had previously taken graduate work elsewhere, from which it appears that there is relatively little migration after graduate status has been attained.

V. *Courses of Instruction.*

Before considering specifically the nature and extent of the courses offered, we will briefly indicate the limitation of their field. Only two institutions have separate departments for applied as distinguished from pure mathematics. Another assigns applied mathematics to the department of astronomy, but it is more usual to include courses of this kind under the head of physics. Frequently the field of this latter department has no clear demarcation from that of mathematics; in general, courses in mechanics and mathematical physics are offered by both departments, with some resulting duplication. This report will cover only the work specifically listed as mathematics.

Courses beyond the first year in calculus are nearly always

announced under two headings, introductory and advanced, often styled "For Undergraduates and Graduates," and "Primarily for Graduates," respectively. Although, as indicated by their title, courses of the former group are attended largely by undergraduates, a considerable proportion of the graduate enrollment is also to be found here, so that they properly form part of the field of the present report.

The American university is apt to be distinctly liberal in its announcements. Thus we find institutions with relatively small equipment offering from fifteen to twenty-five courses in the two groups, and in fact the average for the twenty-four mentioned is about twenty. It should be added that in many cases it is stated that these courses are offered in alternate years, but too frequently the lists are padded and no indication is to be found as to exactly what work was done in a given year. Something less than half the courses announced seem to have been conducted in 1908–1909, about one third of these being of the first group. We may note that the number of courses was thus about the same as that of graduate students, so that the average enrollment per class must have been very small.

The figures that have just been given clearly indicate the ambition of American universities to become centers of graduate study, and raise the question whether in many cases too much is not attempted on too little capital. It would seem that a concentration of advanced work in a dozen or so institutions would be a distinct gain, and that economy in the number of courses might in many cases be advantageously practiced. Often a course is given, not because it is needed, but in order that certain members of the department may participate in graduate instruction. There are, however, several institutions among the twenty-four mentioned which wisely limit their work to a fairly complete list of courses in the first group, with but four or five each year of the more advanced sort. In such cases an adequate training for the master's degree is aimed at, but students who wish to go further are urged to continue their work at one of the larger centers of graduate instruction.

Naturally there is much diversity in the classification of subjects under the two groups; thus we might expect Harvard, which encourages an unusual amount of specialization in the undergraduate program, to include in the first group such courses as quaternions and vector analysis, infinite series, special topics in higher algebra, and theory of numbers. Dif-

ferential geometry is similarly classed at Yale, Michigan, and Wisconsin, as well as one or two other institutions, while about the same number place in this group a first course on the theory of functions.

As already indicated, much of the work in applied mathematics is left to the department of physics. This is particularly true of introductory courses. Only nine institutions of twenty-three whose practice was ascertainable announced under the head of mathematics a first course in analytic mechanics. Eight offered a second course in mechanics under the first group, but in only five cases was this given. It is, perhaps, more to be expected that celestial mechanics should be offered as a mathematical subject in but two institutions.

There is substantial agreement in classing the following subjects under the first group: advanced calculus, advanced analytic or projective geometry, solid analytic geometry, determinants and the theory of equations, and an introduction to differential equations. The first two are almost always full year courses while the others are more often of half that length. The extent to which these may be considered standard subjects is indicated by the fact that but one institution on the list fails to offer a course in advanced calculus, and in only one other instance was such a course not given, if we accept the evidence of catalogues; and practically the same is true of the second named subject. Three failed to offer the third and the same number the fourth as separate courses in the group, though we may suppose the subjects to have been included elsewhere, while the last was everywhere listed, either by itself, or in a few cases as the second half of the course in advanced calculus; and in only a few additional cases were these courses not actually given.

Though there is thus seen to be a considerable uniformity as to introductory subjects, it is interesting to note some of the variations from the more usual lists. Five courses in the history of mathematics are announced, three in probabilities and one in finite differences, six in statistics and insurance, and one in the theory of envelopes. Harvard and Iowa combine algebra with projective geometry, and as noted above, in some cases the first course in differential equations is given as a part of advanced calculus. There would thus seem to be evidence of a tendency to consolidate the introductory work into as compact a group as possible, and this tendency is probably on the increase.

The establishment of special honors courses similar to those of Oxford and Cambridge has been much discussed of late, and has been recommended in recent reports of several university presidents, but so far Princeton and Columbia alone have adopted such a system. This movement, though of great interest, will not be further described here as it at present concerns undergraduates only.

In the consideration of the group of advanced courses it will be useful for some purposes to introduce what may appear to be an arbitrary distinction in classing institutions; those which gave more than six courses of this kind will be placed together, while the others will be regarded as constituting another division. As a matter of fact, however, this corresponds to a real difference in the character of the work undertaken; the thirteen universities of the former class all have candidates for the doctor's degree among their students, while this is true of almost none of the others. As might be expected, a majority of the institutions giving more than six advanced courses are to be found among the older universities of the east.

As to subject matter, we may note that nearly everywhere far more emphasis is placed upon analysis than upon algebra, geometry, or applied mathematics. In but one institution of the first division are more courses given in any one of the last three subjects than in analysis, and the same is true of the class enrollment. As to the relative attention paid to the other branches, this varies widely, depending chiefly on the special interests of members of the department. Only two institutions of the first division leave all work in applied mathematics to other departments, and only two fail to offer courses in both geometry and algebra. We may conclude that there is generally a conscious effort to give a balanced curriculum, with especial emphasis on such subjects as the teaching staff is best prepared to offer. With institutions of the second division an even distribution of each year's work is, of course, impossible. Most of them, however, vary the courses given from year to year so as to include as wide a field as possible. In a few cases this seems to be only a matter of intention, and the courses actually conducted are confined to the subjects most interesting to the two or three men who have charge of graduate instruction.

All the universities of the first division include in their program from two to seven research courses or seminars, while

none of the others has more than one. As this matter lies in the field of another committee, it will not be further discussed in this report.

Both in institutions of larger and of smaller equipment the practice of varying the program from year to year prevails. This is obviously a necessity where the enrollment is small. A number of the universities of the second division make it a rule never to repeat an advanced course in successive years, and this seems also to be true of at least three of the first group. In only two or three institutions are there more than four courses repeated year after year, and even in these cases the subject matter is seldom exactly the same. By far the most usual method of announcing courses consists in grouping them in pairs, sometimes from the same field but often widely differing in nature, offered by the same instructor in alternate years. Several institutions announce all of their work in this way. In a few cases more extended sequences are offered in which the cycle consists of more than two years. The effort thus shown to give variety to the work of the teaching staff and to cover as broad a field as possible is in the main most commendable, but it sometimes leads to the giving of advanced courses by men who may be qualified to do work in certain lines but are obviously unprepared for what they undertake.

Although there is a marked tendency not to devote whole courses to special topics, and one may note a certain conservatism in choice of subjects, most institutions offer at least one course in some comparatively narrow field, frequently one of recent development. Thus we find integral equations (given by three institutions) as well as point sets, divergent series, functional equations, modern theories in geometry, general analysis, differential invariants, linear systems of algebraic curves, imaginaries in geometry, symmetric functions, elasticity, and capillarity. Advanced courses in insurance, probability, and statistics appear in three announcements. In two instances the influence of Sylvester probably accounts for such subjects as the logic of mathematics, and principles of mathematics.

Although this may encroach somewhat on the field of another committee, we may at least mention here the courses offered specifically on the teaching of mathematics. As might be expected, these differ widely in nature. We shall not here enter into their character, but merely report that all but seven institutions of the list offer work of this kind.

This section should not be concluded without some reference to the courses of lectures delivered by distinguished foreign mathematicians on various occasions within the last thirty years. It is true that comparatively few graduate students heard them, but the influence of Cayley's course has already been indicated, and the visit of Klein in 1893 was probably no less important. Of late a number of such courses have been given, and the exchange system may possibly lead to further developments along this line.

VI. *Methods of Instruction.*

Practically all the work, both introductory and advanced, considered in the preceding section, is given by means of lectures of a little less than an hour in length. Sometimes seminars or research courses meet for two consecutive hours. By far the most usual program consists of three lectures a week, but three or four institutions prefer two-hour courses; very few classes meet but once a week, or oftener than three times.

A questionnaire on methods of instruction was sent to eighteen of the twenty-four institutions previously mentioned and replies were received from the following: Bryn Mawr, California, Chicago, Columbia, Cornell, Harvard, Illinois, Michigan, Missouri, Northwestern, Pennsylvania, Princeton, Wisconsin, and Yale. It appears that on some points there is a considerable diversity of practice, even among instructors in the same institutions, for freedom of instruction is, fortunately, almost everywhere carefully preserved. However, in most cases the answers received stated that the department used substantially uniform methods.

The first question concerned the assigning of courses to the members of the department, and was put in the form, — Are standard courses (the theory of functions, for example) given by the same or by different instructors each year? Only two answers indicated that the latter alternative was the rule. Two more, however, stated that a change in this direction was probable, while four indicated some variation in practice. The custom of permanently assigning certain fields to certain men thus seems to be distinctly in favor. In spite of the advantages of such a system it is undoubtedly true that in many cases it is carried too far, so that a subject comes to be considsidered a particular instructor's preserve, not to be trespassed upon.

An inquiry into the matter of examinations reveals two

widely differing conceptions of graduate work. On the one hand the practice of Harvard, for instance, in requiring searching written examinations at the end of each semester indicates a general system of careful supervision, while Columbia, Princeton, Chicago, Bryn Mawr, and California, which, it seems, ordinarily give no course examinations, represent the opposite extreme where the student may take or leave the intellectual repast spread before him. Two or three other institutions allow oral examinations, and in some cases there is a difference of rigidity in the conduct of introductory and advanced courses. It appears that Yale, Harvard, Wisconsin, Northwestern, and Missouri represent the maximum of strictness in requiring written semester examinations. It is hard to determine any tendencies in this matter.

Another question that has a similar bearing concerns the giving of grades indicating excellence of work, a custom which prevails in undergraduate courses in American universities. The answers received show that the opposite practice is most usual in graduate work, but Harvard, Wisconsin, Chicago, Illinois, and Northwestern report that such grades are given.

To the query, — Are problems assigned for report from time to time? — an almost universal affirmative was returned, so that it would seem incorrect to conclude that anywhere is the student expected to be merely a hearer of lectures. It appears that in some courses in Columbia and Chicago this may have been the case, but the universal rule elsewhere is to require reports of some kind on assigned work. On the other hand stipulated reading in connection with courses is almost never insisted on. Lists of authorities and references are assigned but the responsibility for consulting these is thrown upon the student. Only three replies indicated an occasional exception to this rule, while but one or two stated that reports are required on such outside reading.

As already said, the work is given almost entirely in lectures, though only five replies indicated that text-books were never used. However, it would seem that they are followed in only a few introductory courses, and that even here they are used rather as reference books. Perhaps the most usual exception is in the first course in differential equations. The replies received were unanimous in stating that there were no courses in which a text-book was used exclusively, no lectures being given.

It is customary to devote some hours, or parts of hours, to reports of students and discussion of lectures. However, at Harvard reports are given in seminars only, while at Princeton and Northwestern they are very infrequent. Yale, California, and Cornell do not give class time to discussion of lectures, though the last named appoints regular hours for individual consultation. At Wisconsin oral quizzes on lectures are a feature. The most usual practice everywhere is to allow a few minutes of each hour for discussion, questions being frequently asked in interruption of a lecture. The instructor's procedure is as a rule quite informal. In some cases an hour is reserved once a week or twice a month for discussion and reports.

In addition to the regular lecture and seminar courses, some institutions maintain courses for reading and research, in which the student (usually there is but one) meets the instructor at intervals for report and consultation, but no stated lectures are given. Often, but not always, this is done in connection with the writing of a thesis. Harvard announced eight such courses last year, Cornell two, Michigan two, and Missouri one. In the last two cases only were written examinations given, but in all but two courses problems were assigned for report.

Work in absentia has decreased almost to the vanishing point, Chicago alone conducting a few graduate courses by correspondence.

A number of the larger institutions are well supplied with models, calculating machines, and other mathematical apparatus, but others are more meagerly equipped. In several cases the making of models has had a place in graduate instruction.

A special feature at Yale has been the writing up of lecture notes by students and placing them for reference in the seminar room. This practice has been followed elsewhere for certain courses, but does not seem to be the rule in any other institution.

In the main it would appear that little change in methods is contemplated, and certainly no general movement is on foot. Two or three institutions have inaugurated considerable departures from past usage, but this is in each case an individual matter due to changes in the personnel of the department.

VII. *Requirements for the Master's Degree.*

The master's degree "in course" is given by practically all American colleges and universities as representing one full

year of graduate study.* The title of Master of Arts or of Science is also given as an honorary distinction under entirely different conditions, but we will here consider only the degree "in course." This is sought especially by students who either have but one year to devote to advanced work or are not sure of being able to continue for the doctorate, since its possession is often a decided advantage in obtaining a teaching position. Even in secondary work the master's degree is frequently insisted on, and this tendency is so marked that in the opinion of many it will before long be as much in demand there as the doctor's degree now is for the candidate for a college position.

Excluding varieties of nomenclature corresponding to technical or professional courses, four master's degrees are given, namely, Master of Arts, Master of Science, Master of Philosophy, and Master of Letters, but of the last two the former is conferred by but three institutions of the twenty-four previously referred to, and the latter by only one. Nearly half give the master's degree in arts only. The distinction is, except in a few instances, an artificial one in that it does not indicate the candidate's field of advanced work, but merely shows which bachelor's degree he has received. It is noteworthy that in eleven institutions of the twenty-four this distinction has been abandoned, and even where the degree in science is preserved it is often possible for any who prefer it to receive the arts degree provided their work is not mainly professional or technical.

As noted on page 236, about sixty master's degrees were conferred on students specializing in mathematics in the year 1908–1909, a number but little less than one-fourth the total graduate enrollment in that subject. Of the twenty-four institutions especially considered, only five or six failed to confer the degree in that year. Two institutions, Harvard and Columbia, gave the maximum number, eight each.

The universal condition for admission to candidacy is the possession of a bachelor's degree from an institution of equivalent standing. Although there is some latitude in the interpretation of this rule, it is but fair to say that the student admitted with inferior preparation is usually required to make up his

*Johns Hopkins University seems to be alone in requiring two years. Bryn Mawr gives the degree to no students who have done their undergraduate work elsewhere, except as an accompaniment to the doctor's degree.

deficiencies, taking a longer period than the minimum of one year for obtaining the degree. Applications for candidacy accompanied by a program of studies must usually be filed early in the first semester and must be approved either by a committee of the faculty, or by the dean of the graduate school. In case a thesis is required, the subject must be submitted soon after admission to candidacy. The number of courses which must be carried varies from three to five, but in the latter case an allowance is made for the thesis. Some of this work may be done in summer school, and in a number of institutions the degree may be obtained by attendance on three or four such sessions, with the usual requirements as to examination and thesis. Correspondence study or work in absentia has nearly disappeared so far as mathematics is concerned, though some catalogues still announce that part or all of the requirements may be thus met by students who have previously been in attendance. In a few cases where five three-hour courses must be carried a deduction is made for advanced work taken by the student as an undergraduate, but in excess of the requirements for the bachelor's degree. A combination with professional work is sometimes possible, but in the case of mathematics this must be confined chiefly to teaching and engineering.

In 1904 a committee, after two years' consideration and careful inquiry, reported to the Chicago Section of the American Mathematical Society on the requirements for the master's degree, with especial reference to conditions in western institutions. We will here use their recommendations, which, they state, correspond with the requirements at several leading American universities, as a basis for discussing present conditions. Their findings will, however, be only briefly summarized, since the details are easily accessible.*

The first questions raised are, — What should be the minimum acceptable time devoted to undergraduate mathematics, and how should it be distributed? — The committee agrees that this should be the equivalent of five three-hour courses carried continuously throughout the four years of the undergraduate course, and distributed about as follows: In preparation for the calculus (algebra, trigonometry, and analytic geometry), five year-hours; calculus, including differential equations and mechanics, six hours; advanced algebra and geometry, four hours. It will be noted that this list includes

* Published in the BULLETIN, 2d ser., vol. 10, No. 8 (1904), pp. 380–385.

seven year-hours in courses classed as "introductory" in section V. Courses in applied mathematics are not specifically recommended, though it is urged that undergraduate instruction maintain a close contact with the physical sciences. Except that four undergraduate courses are more usually taken as a minimum, and that the exact distribution here shown is not always insisted on, it may fairly be said that the stronger institutions at present comply very closely with the above conditions.

The next section of the report reads: "Upon the basis of the preparation indicated above, it is the opinion of the committee that the work leading to the master's degree should include at least 270 hours * selected from the following groups of subjects. The selection should in any case include subjects from at least two of the three groups.

I. GEOMETRY.
Projective geometry.
Modern analytic geometry, algebraic curves and surfaces.
Application of calculus to twisted curves and surfaces. (Differential geometry.)
Solid analytic geometry.
Descriptive geometry. (Darstellende Geometrie.)

II. ANALYSIS.
Theory of equations.
Advanced calculus.
Theory of functions.
Differential equations.
Theory of numbers.
Invariants.

III. APPLIED MATHEMATICS.
Analytic mechanics.
Mathematical astronomy.
Mathematical physics.
Mathematical theory of probability.

"This arrangement provides that the candidate, in case he enters upon his graduate work with the minimum mathematical preparation indicated above, and hence having a broader *general* training in his undergraduate course, should be required to take at least two thirds of his year's graduate work in pure and

* The term hour here refers to a single lecture hour, so that the above is equivalent to two and one half three-hour year courses.

applied mathematics. On the other hand, as it may be assumed that the student entering upon his graduate work with a larger mathematical credit than this has had less *general* training in his undergraduate course, it is the opinion of the committee that he should be permitted to take as much as one half of his work in departments other than mathematical, providing, however, that the minimum requirement in mathematics indicated above is also fulfilled."

Present practice tends to reduce somewhat the minimum here recommended, so that in a majority of the larger universities two courses in mathematics are accepted, though as a matter of fact more are usually taken. Four institutions require a candidate to present three subjects and two others require work in at least one additional department. The majority would seem to permit the student to take all his work in mathematics if he so chooses, but with one or two exceptions there seems to be no institution where this is distinctly encouraged. No general rule obtains as to the distribution of the candidate's work; this is nearly always an individual matter decided by the instructor or committee in charge. It is probable, however, that the recommendation presented above represents nearly the average of practice.

An interesting variation from ordinary requirements is presented by the University of Michigan. In addition to "Plan A," which calls for the usual amount of specialization, a scheme styled "Plan B" allows a wider choice of studies. Here the thirteen hours to be taken may be distributed as is seen fit among three departments, and other requirements would seem to be less rigid, but the year's work will not usually count as one of the two or three necessary for the doctor's degree.

It is apparently nowhere stipulated, except at Bryn Mawr, where unusual conditions obtain (see footnote, page 245), that the candidate be able to read a foreign language. The report under consideration recommends a knowledge of German and considers French also desirable. As a matter of fact the requirements for the bachelor's degree are such that nearly every graduate has had at least one year's work in one of these languages, but seldom enough to make reading easy. Nevertheless, in most cases some reading of foreign mathematical texts is urged, though rarely required.

The committee quoted believe that all candidates should present a thesis, but that this need not embody original research.

It appears that thirteen institutions of the twenty-four adhere strictly to this rule, while in at least five more the requirement is optional with the instructor in charge or the head of the department. In three or four others a thesis is never presented for the master's degree.

The last topic considered is that of examinations. The report mentioned recommends either a final test covering all the graduate courses taken by the student, or the requirement that a high grade be obtained in term examinations in each course. The former alternative is the more usual one, — in fact it obtains at eighteen of the twenty-four institutions considered, while in several of the others either procedure is possible. At Harvard one of the two highest of the four passing grades in each course is the sole requirement. The final examination may be either oral or written, — practice seems to be fairly evenly divided in this matter, — and if oral, it is usually conducted by a committee of instructors from the departments involved.

There are smaller and less reputable institutions in which the requirements are lower, and in fact some where all requirements are merely nominal. The present report has aimed only to present the state of affairs in the institutions which are really prepared to confer the degree.

GEOMETRIC TRANSFORMATIONS.

Geometrische Transformationen. Zweiter Teil: die quadratischen und höheren birationalen Punkttransformationen. By KARL DOEHLEMANN, associate professor of mathematics at the University of Munich. Leipzig, Göschen (Sammlung Schubert, XXVIII), 1908. viii + 328 pages and 59 figures.*

AT the time this volume was written there was no book devoted to the subject of its title. The next year appeared the fourth volume of Professor Sturm's treatise † and several parts of the Encyklopädie, so that now we are fairly well supplied with an introduction to this most fundamental study. And the two books do not overlap; the larger work is synthetic and replete with geometric illustrations, while that under review is

* The first part of Professor Doehlemann's treatise appeared in 1902; it was reviewed in the BULLETIN by Professor Gale (vol. 10, pp. 512-515).

† See the review in the BULLETIN, volume 17 (1910), pp. 150-154.

analytic, elementary, and well adapted to its purpose of serving as a finder into the literature and problems of birational transformations. Perhaps it would have been well to have the name Cremona transformations in the title, for no mention is made of the geometry on a given curve or surface, hence such concepts as point groups, series, residuation, adjoint curves, etc., are entirely absent, as is also the entire theory of multiple correspondences and higher involutions. The selection of a restricted field is a wise one; to treat adequately the subject of birational transformations in this broader sense requires several volumes, and goes beyond the task imposed in the Schubert collection. In fact it is the introduction that is particularly needed. If a reader has a thorough knowledge of Cremona transformations, he is well prepared to study the geometry on a curve or surface by the Brill-Noether method.

The book is divided into two nearly equal parts, the first treating of the plane, the second of space. The first chapter introduces quadratic transformations by means of two pairs of projective pencils of lines, first synthetically, then algebraically. The transformation of a given curve is discussed for a large number of special cases, including the images of an extensive list of singular points. This part is developed synthetically only, thus not furnishing the reader with complete weapons for the resolution of the singularities of a curve whose equation is given. The general bilinear equations are now shown to define the same transformations; the cases in which two or three of the vertices of the fundamental triangle approach coincidence being briefly considered. Finally, the case where the fundamental triangles of the two planes coincide and the condition for involution are discussed. At the end of this and of each subsequent chapter is a list of the memoirs bearing on the topics considered. These lists are not complete, and a few conspicuous absences are noticed, but on the whole they are well selected and give a very good idea of the literature and of the unsolved problems.

Now follow two chapters (pages 50–134) on metrical properties based upon the case in which two of the vertices of the fundamental triangle are the circular points at infinity. Foci, minimum lines, reciprocal radii, orthogonal circles, and analagmatic curves are discussed in a very elementary manner, most concepts being developed independently, then shown to belong under the preceding projective treatment. A number of applications to mechanics, inverting straight lines into circles, are

considered, and a number of properties of bicircular quartics and of circular cubics are given. The second of these chapters considers the bilinear relation between two complex variables, the development being similar to that given in most books on the theory of functions. It includes conformal mapping and gives a number of elementary examples. While it is interesting to note the position of this transformation as a particular case of those mentioned before, it seems hardly justifiable to devote over one tenth of a book concerned with geometric transformations to the discussion of a case whose main use is in analytic problems, and which has been treated repeatedly elsewhere. This, however, is the only case in which the perspective of relative importance (for geometric purposes) does not seem to be preserved in the best way. Even here it might be said that all the steps of the general case can be understood most easily by a careful study of the simple one.

Chapter IV, on the general birational transformation of the plane, commences from an entirely different point of view. If

$$x_i' = \phi_i(x_1, x_2, x_3)$$

defines a rational function of the x_i, under what conditions can the equations be solved rationally for x_i in terms of the x_i'? In the search for the answer to this problem we naturally find the Cremona net, meet with fundamental curves and fundamental points, and discover a number of relations existing among them. The Jacobian of the Cremona net is shown to consist of all the fundamental curves of the transformation. A table of all the forms up to $n = 10$ is given, n being the order of the rational curve, image of a straight line. The theorem that every Cremona transformation can be resolved into quadratic inversions is proved in detail when the basis points are distinct; mention is made of the modifications necessary when two or more of these approach coincidence, and the memoirs treating of these cases are cited.

The first part closes with a rapid glance over the newer problems, most of the theorems being given without proof. Since the name of the author appears frequently in the literature of these sections, he is particularly well fitted to prepare such a survey. The involutorial transformations are discussed in some detail, in particular the Jonquières transformations. The results of the researches of Bertini regarding the number of types of irreducible involutions are given, and a method of

obtaining all except that of order 17 is added. A section on periodic transformations gives a résumé of the results of Kantor, discusses conjugate and commutative transformations but does not give any geometric criteria. The concept of finite groups of Cremona transformations makes still further demands on the algebra of substitution groups; the results obtained by Wiman are given without proof. A short statement concerning continuous groups is also made, principally to refer to the literature. Whether there are curves which remain invariant under an infinite discontinuous group is not mentioned.

In the second part, the quadratic transformations of space are introduced by means of two reciprocal systems. The treatment is synthetic, and includes a discussion of fundamental elements and of the image of a general curve and a general surface. Then follows an algebraic development of the same idea, the two together making a symmetric and well balanced treatment. Particular cases are discussed briefly but not exhaustively. One misses the inverse of the general quadratic transformation. It is introduced later, but here the reader meets the sole statement that the inverse of a transformation may be of different order than that of the direct operation. The generalized stereographic projection and the statement that the image of a plane is a surface which can be rationally mapped upon a plane fittingly close the chapter.

Now follow two chapters (pages 201–286) on the particular case in which the fundamental conic is the absolute circle at infinity. As in the plane, the discussion is very elementary and proceeds algebraically with non-homogeneous coordinates. Besides the geometry of reciprocal radii a number of applications are made, particularly to the simple stereographic projection and Mercator's chart. Then follows a systematic geometry of the sphere, including complexes, congruences, depiction into hyperspace, and the generation of Dupin's cyclides. Thus far everything is linear; the discussion of the quadratic equation is much more brief. No mention is made of the higher geometry of the sphere, which contains perhaps the most elegant example of birational transformation in the whole field. The excuse may be made that in the elementary orthogonal case we are still dealing with a point transformation, whereas in the general case we are not. On the other hand the entire treatment is in terms of the sphere as element of space, and its coordinates are interpreted as point coordinates in hyperspace.

No apology need be made for this long digression into the geometry of the sphere. Here the purpose is a geometric one and the discussion keeps well within the limits set by the book. The surfaces which naturally appear in this way can be interpreted projectively. If, finally, the line-sphere transformation be applied, we have the Kummer surface and as particular cases the Steiner surface and the quartic ruled surfaces. Together they comprise practically all the quartic surfaces that we know anything about. A short section on electric images contains a list of most of the important memoirs devoted to it.

Chapter VIII, on cubic and higher birational transformations of space, begins with three bilinear equations analogous to the pair previously treated in the plane. It is exceptionally well written and contains all the important principles for the discussion of the (3, 3) transformation, that is, that in which planes go into cubic surfaces, and the inverse is of the same form. By means of these principles it is easy and natural to discuss the mapping of a general cubic surface on the plane. In the treatise of Sturm this depiction precedes the consideration of the cubic transformation, but the treatment of our author seems more natural and is much easier to follow. The details are not given, but three different depictions are outlined.

The last section contains a brief treatment of the general birational transformation, including a discussion of the fundamental points, curves, and surfaces. The monoidal case being given as an example, a sketch is added regarding the method of procedure in the determination of new transformations, and in the corresponding depiction of a surface upon a plane.

Of the few typographical errors noted the only ones that may cause confusion are found on pages 140 and 141. The letter i is there used as symbol of summation in two different senses, the first running over the three coordinates, and the other over the h fundamental points of the plane. In the resulting formulas these two symbols are curiously combined.

In the book under review Professor Doehlemann has admirably fulfilled the purpose aimed at in the Sammlung Schubert.

VIRGIL SNYDER.

SHORTER NOTICES.

The Collected Mathematical Papers of James Joseph Sylvester. (Edited by H. F. BAKER.) Volume III. xv + 688 pp. Cambridge, University Press, 1909.

THE first two volumes of Sylvester's works appeared in 1904 and 1908, and were reviewed by the present writer in the BULLETIN, volume 15 (1909), pages 232–239. The third volume contains chiefly the papers written while Sylvester was at the Johns Hopkins University. The majority of these papers relate to the theory of invariants, in particular with the use of generating functions to enumerate the complete system of concomitants of a system of quantics. In two of the earlier papers of this series Sylvester proved the dependence of two of the thirty fundamental forms of two binary quartics given by Gordan, and emphasized the importance of the English enumerative method in questions on the dependence of the invariants. Among the longer papers is that "On an application of the new atomic theory to the graphical representation of the invariants and covariants of binary quantics." On the origin of this paper on chemistry and algebra, Sylvester says: "Casting about, as I lay awake in bed one night, to discover some means of conveying an intelligible conception of the objects of modern algebra to a mixed society, mainly composed of physicists, chemists, and biologists, interspersed only with a few mathematicians, to whom I stood engaged to give some account of my recent researches in this subject of my predilection, and impressed as I had long been with a feeling of affinity if not identity of object between the inquiry into compound radicals and the search for 'Grundformen' or irreducible invariants, I was agreeably surprised to find, of a sudden, distinctly pictured on my mental retina a chemico-graphical image serving to embody and illustrate the relations of these derived algebraical forms to their primitives and to each other which would perfectly accomplish the object I had in view."

Mention should be made of the elaborate memoir on ternary cubic-form equations in regard to their solvability in rational numbers, in particular the cases in which an integer can be expressed as a sum of two cubes. The paper also treats of the arithmetical properties of cyclotomic functions and of polygons which can be inscribed and escribed about a cubic. The volume contains various short papers on the theory of numbers, in

particular one on the totality of primes comprised within given limits.

The memoir on subinvariants and perpetuants belongs to this period. At the end of the volume appear some short papers on the theory of partitions; but Sylvester's chief work in this field will appear in the fourth (final) volume of his collected papers.

L. E. DICKSON.

Niedere Zahlentheorie. Zweiter Teil. By DR. P. BACHMANN. B. G. Teubner's Sammlung von Lehrbüchern auf dem Gebiete der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen. Band X, 2. Leipzig, 1910. x + 480 pp.

THE first volume of this work appeared in 1902 and was reviewed by the writer in the BULLETIN, volume 9 (1903), page 555. The present second volume has the subtitle Additive Zahlentheorie, a term suggested by Kronecker for the properties of numbers relating to their additive combinations.

The first chapter deals with arithmetical series of the nth order, polygonal and figurate numbers, the sum of the kth powers of the first n integers, the sum of the kth powers of n numbers in arithmetical progression, and at length with the theory of Bernoullian numbers. The second chapter deals with recurring series, in particular with those bearing the names Farey, Fibonacci, Fermat, Pell, and Dupré. Application is made to factorization of numbers of certain forms, perfect numbers, Fermat and Mersenne numbers. Here results are omitted that have been known for several years. For instance the number $2^{67} - 1$ is left in doubt although Cole has given two factors (BULLETIN, 1903, page 137). Chapters III–V relate to the theory of partitions. Chapter VI is entitled recursion formulæ and deals with various number theoretic functions. Chapters VII–VIII treat of the representation of a given number as a sum of like powers or as a sum of multiples of powers. Every positive integer can be expressed as the sum of four squares (but not always as the sum of three); as the sum of nine cubes (but not always as the sum of eight); as the sum of 37 fourth powers (though doubtless this limit is too high). Further theorems relate to the number of representations of an integer as the sum of four squares. The related investigations by Liouville are given at length. The final

chapter deals with Fermat's equation $x^n + y^n = z^n$ from the standpoint of the elementary theory of numbers, giving Fermat's proof for $n = 4$, the Euler-Legendre proof for $n = 3$, and remarks on the Dirichlet-Legendre proof for $n = 5$, and the Lamé-Lebesgue proof for $n = 7$. Kummer's method by ideals is beyond the scope of the work; the comment on regular primes (page 461) is corrected at the end of the book. The formulas obtained independently by Abel and Legendre are established. The developments by Sophie Germain, E. Wendt, and L. E. Dickson are then cited. In one instance (page 475), the initials of the last name are given incorrectly; while, in a quotation from Sylvester on page 104, permeating is spelled wrong. However, the book is especially free of errata and the typography is excellent. In the present text Bachmann has fully maintained his reputation as to clearness, thoroughness, and exhaustiveness.

L. E. DICKSON.

Eléments de Calcul vectoriel avec de nombreuses Applications à la Géométrie, à la Mécanique et à la Physique mathématique. Par C. BURALI-FORTI et R. MARCOLONGO. Traduit de l'italien par S. LATTÈS. Paris, A. Hermann et Fils, 1910. vii + 229 pp.

So lengthy a review * was recently accorded to two new books on vector analysis by Burali-Forti and Marcolongo that nothing more than the mere mention of the French edition of the first of the two would be needed, were it not for the fact that in the French the authors have added a long and excellent appendix on Grassmann's geometric forms and on Hamilton's quaternions. The object of the appendix is to show the power of the authors' vector analysis by using it to set up the Grassmannian and Hamiltonian systems. There is apparently the further object to set forth these two mathematical disciplines in such a way that mathematicians in general, and in particular those mathematicians who think they know something about the systems, shall be led to conceive or reconceive, as the case may be, these systems as they should be conceived. We have no exceptions to take to the authors' presentation of the subject; it is compelling.

There is one remark, found on page 201, which deserves

* Under the title "The unification of vectorial notations," BULLETIN, vol. 16, pp. 415–436 (May, 1910).

particular attention. The authors have defined quaternions in their way and have given Hamilton's definition and then remark that they will prove that the quaternions given by their definition are the quaternions of Hamilton and not any other kind of quaternions. The importance of this remark is the implication that there are other quaternions than Hamilton's. Of course the authors are not interested in any others any more than they seem to be interested in Gibbs's algebraic point of view; but we are glad to have them imply that there are others — for in the strictest sense it is precisely other quaternions than Hamilton's original ones that most quaternionists now use. Gibbs * pointed out one defection from the strict Hamiltonian point of view; the authors have frequently pointed out others for the purposes of berating the quaternionists who use them.

It may interest some, who had not the opportunity to hear Gibbs, to see (as well as I can remember) the way in which he presented his, not Hamilton's, quaternions. He considered ordinary vectors $\alpha, \beta, \cdots$ between which he had already defined his scalar product $\alpha \cdot \beta$, his vector product $\alpha \times \beta$, and his dyad product $\alpha\beta$; and he proceeded to define his quaternion product as †

$$\alpha \circ \beta = -\alpha \cdot \beta + \alpha \times \beta.$$

Thus his quaternion was the sum of a vector and a scalar — which Hamilton's quaternion primarily was not. In particular, scalars and vectors were merely especially simple quaternions. The defining equation gave at once

$$i \circ i = j \circ j = k \circ k = i \circ j \circ k = -1$$

and all the formal laws of quaternion algebra. Here i, j, k were vectors and the product was quaternionic. In Hamilton's analysis the i, j, k would be not vectors but right quaternions, and for him the right quaternion was not primarily a vector. So far as we can judge, the authors are entirely right about Hamilton, and the followers of Hamilton are a good deal wrong about him. They do not any longer use the original quaternions; they use their own, which are practically identical with Gibbs's; it is a pity the authors have not analyzed these that are used instead of the original ones which Hamilton analyzed with such detail. E. B. WILSON.

* The Scientific Papers of J. Willard Gibbs, vol. 2, p. 172.

† He did not use the sign ∘ or any other special sign for the quaternion product.

Gaston Darboux. Biographie, Bibliographie analytique des Ecrits. Par ERNEST LEBON. Paris, Gauthier-Villars, 1910. viii + 72 pp.

THIS is the second volume of the series Savants du Jour, which M. Lebon is bringing out. It is as regards content and arrangement similar to its predecessor, which was devoted to the life and work of Poincaré.* It begins with a short biography of Darboux (7 pages). A few extracts may be of interest to the readers of the BULLETIN. Darboux was born at Nîmes, August 13, 1842, the elder of two sons of a grocer. His father died in 1849. But his mother, noting the intellectual gifts of her two sons, provided for their education. In 1861 the youthful Gaston was awarded the rare distinction of being admitted as the first among all candidates both to the Ecole Polytechnique and to the Ecole Normale. He chose the latter. He received his doctor's degree from the University of Paris in 1866. After several years spent in secondary teaching, during which period he published several important papers, he became in 1872 "maitre de conférences" in the Ecole Normale. In 1875, at the age of 32 years, he was awarded the Prix Poncelet by the French Academy for the ensemble of his mathematical work. Since 1880 he has occupied the chair of geometry at the Sorbonne, being the successor in this chair of Chasles. He was elected a member of the Academy of Sciences in 1884, and made a permanent secretary of this body in 1900. He is a member of 21 academies and of 11 scientific societies, and is an honorary doctor of the Universities of Cambridge, Christiania, and Heidelberg.

The remainder of the little volume is devoted to an appreciation of the scientific work of Darboux taken from various sources, and a complete list of his writings. The latter numbers 419 titles, many of which are accompanied by brief abstracts. The amount and quality of such productivity is at once the inspiration and the despair of his would-be disciples. The volume also contains a fine portrait of Darboux.

J. W. YOUNG.

Handbuch des mathematischen Unterrichts. Von W. KILLING und H. HOVESTADT. Band I. Teubner, Leipzig und Berlin, 1910. viii + 442 pp.

NOTWITHSTANDING the great number of handbooks on mathematical instruction the authors justify the present work by the

* Cf. review in the present volume of the BULLETIN, p. 42.

claim that such books in general are too abstract or are too much concerned with methods of teaching and do not embody a sufficiently thorough and painstaking investigation of those things which we try to teach. Hence, assuming that the average person who proposes to teach elementary mathematics has neither the leisure nor means to investigate the advance of the science from original sources, the authors devote at least half of the book to presenting the facts in available form for the novice. The aim is obviously to rid the mind of the teacher of the idea that Euclid is sacred, and to fill him with the modern spirit.

It is worth noting that there are practically no drawings in this volume, devoted to geometry, but the required figures are very carefully described and relations of position fully defined. The reason is obvious, for a single figure is of necessity special, whereas if the reader draws a figure from the description, he will find the latter fits any case.

The book naturally divides itself into two parts. The first half is devoted to a careful and well balanced discussion of the foundations of geometry. The authors define a complete system of axioms and proceed to investigate those of Euclid and Hilbert, the latter being assumed to be the best of modern systems. There is a most interesting general discussion of euclidean and non-euclidean geometries, showing how the latter were forced upon certain investigators against their will. The bibliography here as throughout the book is excellent and is one of its most valuable features. The distinctive characteristics of the various non-euclidean geometries are clearly presented.

In order to show the basal ideas in geometry there is an attractive discussion of natural and non-natural geometries. The futility of *defining* a line as "the locus generated by a moving point," and of similar proceedings is pointed out with some degree of humor. The ideas of right and left on the line and plane, of congruence and symmetry, of the equality of area or content in plane figures, and corresponding ideas in space are treated thoroughly. The chapter on non-real configurations of the plane contains some excellent hints for the teacher presenting this subject. The authors express the opinion that most elementary teachers are unable to comprehend and explain the exact nature of the so-called infinitely distant point. Such teachers would do well to read this vigorous treatment. The chapter on plane construction problems is no less inter-

esting. The problem of determining whether a construction is possible by means of a straight edge and compass is naturally placed at the head of the chapter. Numerous constructions are proposed, including a list of apparently simple ones which are however impossible. Constructions possible by means of other instruments are discussed also, and their relation to the first problem explained.

The two chapters on the logic of geometry and on literary form in mathematical instruction are interesting. By a complete system of axioms, we mean one that contains all specifically geometrical ideas, and we fail to notice the axioms of logic for combining them, which we nevertheless tacitly assume. The authors attempt to build up a system of axioms of this logic. The teacher has further a right to demand good literary style as well as logical precision, but the authors deride hair splitting niceties and cranky insistence on logical and verbal completeness, which only serve to kill the student's interest without clarifying his ideas. Clinch the argument by conversation about the theorem, is their advice. The final chapter of the first half is devoted to a rational discussion of the teaching of elementary geometry. The absolute folly of having pupils commit refined definitions which have no real meaning to them is dwelt upon. In trying to aid the teacher both in methods of instruction and arrangement of material, this book assumes two objects in teaching — to give, first, an insight into the peculiar character of mathematics as a unique domain of human knowledge; second, an understanding of the important rôle it plays because of its numerous practical applications.

The purpose of the second half of the book is to orient the reader as to the essentials, both as to material and as to its arrangement and treatment. No *ideal* course is prescribed, as a course must have personal coloring to be stimulating. But great care is taken everywhere to show the true importance of individual theorems, and give the reader the proper perspective to enable him to choose wisely. Beginning with the triangle, some of the discussion may seem trivial, because we have prejudiced minds as to the proper order and methods. But the succeeding chapter on the theory of parallels is extremely clear, and reveals the thoroughness of the writers. Various methods of approaching the study of the circle are criticised, also that of the equality of plane figures. The teacher must finally answer the double question: What shall I

teach and in what order? A very useful but rather out of place chapter on the theory of irrational numbers is interjected at this point. The theory and application of the Dedekind cut as well as Cantor's theory of irrational numbers are outlined quite fully. Strenuous objection is made to the custom of American texts of applying directly the laws of arithmetical proportions to geometrical quantities. Rational advice is also given against the tendency to number theorems and operations one, two, three, etc., and to compel students to commit this enumeration.

The last seven chapters treat topics rarely found in an elementary American text. The discussion of similarity of plane figures, with some suggested applications, is valuable. The extension of the idea of similarity to circles is very fruitful. Various methods of "measuring" circles are treated, including Huygens' method, although this is pointed out as being of doubtful value in the lower schools. Anharmonic ratios of points on a line and of lines through a point are treated with elegance, and considerable space is devoted to pole and polar with respect to a circle. It seems unfortunate that American texts ignore so largely these fruitful and interesting fields. Pencils and nets of circles, with their properties, are discussed in one chapter and inversion in the plane in another. The last chapter is a long discussion of the Apollonius tac-problem. The methods of Apollonius are first discussed, then the modern solutions of Gergonne, Massfeller, and C. Adams are criticised and tested by Study's criteria. The authors' discussion of Study's criteria for the best solution of a construction problem is not without interest. This last half of the book is replete with brief scientific discussions and helpful remarks which the reviewer must refrain from pointing out in more detail. The authors are not entirely free from some of the faults they inveigh against, but the volume doubtless deserves a place among other books of its kind, for it is fearless in its criticisms of others, is thorough in research, and has the merit of bringing the history and bibliography up to date.

D. D. Leib.

Complementi di Analisi algebrica elementare. By Federico Amodeo. Luigi Pierro, Naples, 1909. 284 pp.

This little volume is one of a series of elementary text books by the same author and is intended for use in the Istituto Technico of Italy. The author says in the preface that a stu-

dent should know the things contained in this book before he enters the university. This makes the volume interesting, as it shows us what is considered a good university preparation in Italy. This is not so different from our own ideas, but many of the subjects here treated we reserve for a course in college algebra. However, many conceptions which we teach in college algebra or at a later period the Italian student gets before he begins his second course in algebra for which the present book is intended. Some of these conceptions are interval, independent variable, inverse function, geometric representation of a function, and sequence of numbers.

The book contains seven chapters the titles of which will sufficiently indicate the contents: Calculus of combinations; Continued fractions; Analysis of indeterminates of the first degree; Inequalities and systems of inequalities; Discussion of equations and problems and equations of second degree; Finite and continuous functions, limits, indeterminate forms; Maxima and minima, discussion of functions. The book also contains an appendix on geometrical conics. Each chapter closes with an excellent set of problems.

Ordinarily the Italian student devotes much more time to the study of elementary mathematics than the American student. But as here indicated, the time is not spent in anticipating college work, as is often done by preparatory schools in this country, but is devoted to doing more thoroughly the ordinary elementary work.

The presentation shows the author to be a master. But the general appearance of the book from the publisher's point of view would not be considered good in this country.

C. L. E. MOORE.

College Algebra. By H. L. RIETZ and A. R. CRATHORNE. New York, Henry Holt, 1909. xiv + 261 pp.

THE introduction to this text is concerned with the reasoning in the transition from numerical to literal quantities. Addition and multiplication are regarded as fundamental operations and no attempt is made to define them. Their laws, including their commutative, associative, and distributive properties, are given as assumptions. Subtraction is then defined with reference to addition, and division with reference to multiplication. The algebraic use of these four operations including the laws of real indices is discussed in some detail in the same chapter. The

student's attention is also called to the fact that proofs are quite different from definitions and assumptions, a distinction which he should understand and appreciate at this stage of his mathematical study.

The second chapter provides for a brief review of the student's previous knowledge of algebra. It contains a number of exercises on algebraic reduction, including parentheses, complex fractions, factoring, and radicals. The application of these principles is illustrated in a list of examples calling for the evaluation of a number of builders' and engineers' formulas. The practical nature of these and many subsequent examples is one of the noticeable and commendable features of the book. Chapters three and four take up functional notation, graphs, and equivalent equations. Following this, determinants of the second and third orders are introduced for the purpose of solving systems of linear equations. A more detailed discussion of the elementary principles of determinants is given near the close of the book.

Chapters six to twelve inclusive deal with quadratic equations, maximum and minimum values of quadratic functions of the form $ax^2 + bx + c$, inequalities, mathematical induction, variation, progressions, and complex numbers. The theory of equations is next taken up in a manner worthy of the student's careful attention. One criticism, however, might be offered concerning the article on Horner's method. The graph of the function is emphasized in a way that leads the reader to think of it more as a means of obtaining results than as an interpretation of them. The latter part of the book discusses logarithms, infinitesimals, limits, series, undetermined coefficients, permutations, combinations, probabilities, and determinants.

While recognizing the book as an excellent one for college work, the reviewer offers a few criticisms. In example 13, page 84, a_m must be positive. The sentence in line 7 from the bottom of page 168 might better be made to read "any *finite* number of variables. The use of the word limit in case (*c*), page 171, is scarcely justified by the author's definition of a limit. The phrase, "If from some point on" in article 135, page 178, is not clear to a person unacquainted with the intended meaning. The expression "-102" in line 4, page 180, might better be omitted; the sentence is complete and expresses the desired meaning more accurately without it. The theorem in article 141, page 188, should read "if each term is

numerically less." The answer given to example 17, page 213, is incorrect. There are a few typographical errors such as $(2n + 1)$ instead of $(2n - 1)$ in example 2, page 87, but they are so trivial as to give no serious trouble and the book as a whole is quite free from them. J. V. McKELVEY.

Das Prinzip der Erhaltung der Energie. Von MAX PLANCK. Zweite Auflage. Leipzig, B. G. Teubner, 1908. xvi + 278 pp.

AMONG the many series of books published under some collective title, one which is at the same time scientific and to a certain extent suited to intelligent popular consumption is Wissenschaft und Hypothese. It is the German translation of Poincaré's La Science et l' Hypothèse, which has the position of volume I and which apparently has given its title to the whole series. Other volumes which have already appeared are Poincaré's La Valeur de la Science (translated), Lipps's Mythenbildung und Erkenntnis, Bonola's Die nichteuklidische Geometrie (from the Italian), G. Darwin's Ebbe und Flut (from the English), Hilbert's Grundlagen der Geometrie, Picard's La Science moderne (translated), and Planck's Das Prinzip der Erhaltung der Energie. From these titles it will appear that the series does not consist chiefly of books written especially for it, but is made up at least in large part of works already written in one language or another and assembled (after translation if need be) into one group. Indeed, some of the volumes were already several years old when the series was started, and among the oldest is Planck's which now celebrates the attainment of its majority with a second edition.

Of such well known and acknowledged value and interest is Planck's discussion of the historical development of the principle of the conservation of energy, of the formation and proof of the principle, and of the different kinds of energy, that little need be said of this second edition, which is in no essential way different from the first. The development of science in the last quarter century has offered much that might be added to the book, but has disclosed nothing that need be taken from it. To this statement one possible exception may be taken in view of Minkowski's recent presentation of electromagnetic and mechanical theories from the point of view of the principle of relativity. In the fundamental matrices which Minkowski sets up, the energy is but one element of many and has no apparent invariance or predominating importance! E. B. WILSON.

Grundriss der Naturphilosophie. Von W. OSTWALD. Leipzig, Philipp Reclam, jun., 1908. 195 pp.

RECLAM'S Universal-Bibliothek is widely known as a good place to look for almost anything from Bret Harte to Kant with tolerable assurance of finding it attractively put up in small volumes at small prices. Probably somewhere between the rather wide limits just assigned would be placed Ostwald's Grundriss der Naturphilosophie; how to place it in much narrower limits is a question; it probably would hardly have appealed to Kant as philosophy or to Bret Harte as nature. There are four parts, general epistemology, logic with the theory of aggregates and mathematics, the physical sciences, and the biological sciences. The point of view throughout is that of the physicist or chemist; the philosophy is not refined, nor the mathematics logical. It is probably precisely these characteristics that will make the book readable and interesting and instructive to the general public. Any admirer of pragmatism will find much here to delight him. To any one not trained in philosophical or mathematical or scientific thinking but desirous of awakening his ideas toward science and philosophy the book is heartily to be recommended as introductory reading.

E. B. WILSON.

NOTES.

The opening (January) number of volume 12 of the *Transactions of the American Mathematical Society* contains the following papers: "An invariantive investigation of irreducible binary modular forms," by L. E. DICKSON; "An application of symbolic methods to the treatment of mean curvatures in hyperspace," by W. H. BATES; "On the order of linear homogeneous groups (fourth paper)," by H. F. BLICHFELDT; "The metrical aspect of the line-sphere transformation," by J. L. COOLIDGE; "Natural systems of trajectories generating families of Lamé," by EDWARD KASNER; "A fundamental system of invariants of the general modular linear group with a solution of the form problem," by L. E. DICKSON; "Linear difference equations and their analytic solutions," by R. D. CARMICHAEL.

Professor C. L. BOUTON has retired from the editorial staff of the *Transactions*. Professors MAX MASON and P. F. SMITH have been added to the staff as associate editors.

THE opening (January) number of volume 33 of the *American Journal of Mathematics* contains the following papers: "On a class of cubic surfaces with curves of the same species," by JOHN EIESLAND; "The automorphic transformations of the binary quartic," by A. H. WILSON; "Theorems on the simple finite polygon and polyhedron," by N. J. LENNES; "On the solutions of certain types of linear differential equations with periodic coefficients," by F. R. MOULTON and W. D. MACMILLAN. The frontispiece is a portrait of Professor ULISSE DINI, of the University of Pisa.

AT the Minneapolis meeting of the American association for the advancement of science it was decided to hold the next meeting of the association at Washington, D. C. Dean C. E. BESSEY, of the University of Nebraska, was elected president of the association. Professor E. B. FROST, director of the Yerkes Observatory was elected vice-president and chairman of Section A. The retiring vice-president, Professor E. W. BROWN, of Yale University, delivered his address on "the relations between Jupiter and the asteroids." The programme of the section consisted of astronomical papers, except at the joint sessions with the Chicago Section of the American Mathematical Society. At the Washington meeting the retiring vice-presidential address will be by Professor E. H. MOORE, of the University of Chicago.

AT the meeting of the London mathematical society held on December 8 the following papers were read: By G. H. HARDY, "Properties of logarithmico-exponential functions," and "Some results concerning the increase of functions defined by an algebraic differential equation of the first degree"; by A. A. ROBB, "Optical geometry of motion"; by T. C. LEWIS, "Note on the Pellian equation," and "A property of the number 7"; by G. B. MATHEWS, "On the arithmetical theory of binary cubic forms."

THE fifteenth meeting of the association of teachers of mathematics of the Middle States and Maryland was held at Philadelphia, November 26. The following papers were read: By

I. J. SCHWATT, "Is the average secondary school pupil able to acquire a thorough knowledge of all the mathematics ordinarily given in those schools?"; by E. H. KOCH, "Training for efficiency in elementary mathematics." Reports by the committee on mathematics in continuation schools and by the committee on a syllabus for algebra were read.

W. H. METZLER was elected president and H. F. HART secretary for the year 1911.

THE ninth regular meeting of the Rochester section of the association of the teachers of mathematics in the Middle States and Maryland was held at Rochester on December 27 in affiliation with the meeting of the New York teachers' association. The following papers were read: By C. W. WATKEYS, "Volumes as functions, with their graphs"; by C. F. WHEELOCK, "Informal discussion of mathematical work in New York State"; by F. W. Owens, "On the superposition postulate in geometry"; by W. BETZ, "Causes of failure in mathematics: report prepared for the international commission."

THE Syracuse section of the association of teachers of mathematics in the Middle States and Maryland held its annual meeting at Syracuse in affiliation with various other educational organizations on Friday, December 30, at which the following papers were read: By W. B. CARVER, "Some topics of school mathematics of especial importance to students who expect to study analytic geometry and calculus"; by C. A. SHAVER, "The new syllabus in arithmetic for New York State"; by W. SMITH, "What results are we getting from graphic algebra?"

THE New York section of the Association of teachers of mathematics in the Middle States and Maryland had a general meeting at the Hotel St. Denis in New York city January 6, 1911. After a dinner served to about 125 guests, the members heard the reports on the progress of the International commission on the teaching of mathematics made by the American commissioners, Professor D. E. SMITH, Professor W. F. OSGOOD and Professor J. W. A. YOUNG.

UNIVERSITY OF ILLINOIS. Courses offered in the second semester, 1910–1911. — By Professor S. W. SHATTUCK: Calculus of variations, three hours. — By Professor E. J. TOWNSEND: Functions of a complex variable, three hours. — By

Professor G. A. MILLER: Elementary theory of groups, three hours. — By Professor H. L. RIETZ: Theory of averages and the mathematical theory of investments, three hours. — By Professor C. H. SISAM: Algebraic surfaces, three hours. — By Professor J. B. SHAW: Vector differentials and integrals, three hours. — By Professor A. EMCH: Elliptic functions, three hours. — By DR. A. R. CRATHORNE: Theory of linear differential equations, three hours. — By Dr. G. E. WAHLIN: Partial differential equations, three hours. — By Dr. R. L. BÖRGER: Projective geometry and linear transformations, three hours.

AT the annual public meeting of the Paris academy of sciences on December 19 the following announcements were made. The grand prize of the mathematical sciences was not awarded, as no meritorious memoir on the stated problem had been received. The Francoeur prize was awarded to E. LEMOINE; the Poncelet prize to M. RIQUIER; the Binoux prize to E. LEBON. The Montyon prize in mechanics was awarded to J. GAULTIER for his inventions in connection with surveying instruments; the Fourneyron prize was not awarded. The Pierre Guzman prize was not awarded, but the accrued interest was granted to the late M. LÉVY; the Lalande prize was awarded to P. H. COWELL and A. CROMMELIN for their researches in connection with Halley's comet; the Valz prize to S. JAVILLE for his work on nebulae and periodic stars; the Janssen medal to W. W. CAMPBELL for his researches in stellar spectroscopy. The Jerome Ponti prize was awarded to H. ANDOYER for his new trigonometric tables in which the logarithms are carried to 14–17 decimal places.

THE annual meeting of the Italian association for the advancement of science was held at Naples, December 15–21, under the presidency of Professor CIAMICIAN, whose address was on "The cooperation of the sciences."

In pure and applied mathematics the following papers were read: By E. BOMPIANI, "Contribution to projective differential geometry of hyperspaces"; by M. O. CORBINO, "Fifty years after the discovery of Pacinotti's ring"; by A. GARBASSO, "The emission of light"; by A. DE NORA, "Remarks on the Müller-Breslau method of the calculation of frameworks"; by L. SILBERSTEIN, "Mutual mass of two electrons"; by C. SOMIGLIANA, "On elastic properties of the

earth"; by A. TUMMARELLO, "Types of homaloidal surfaces"; by G. VACCA, "History of mathematics in the Far East, contributions by T. HAYASHI and V. MIKAMI."

DR. M. CIPOLLA, of the University of Palermo, has been appointed associate professor of algebra at the University of Catania.

DR. E. VESSIOT, of the University of Lyon, will give a course in mathematical analysis at the University of Paris during the second semester of the present year.

PROFESSOR C. NEUMANN, of the University of Leipzig, will retire from active teaching at the end of the present semester. He will complete his seventy-ninth year on May 7.

MR. H. J. PRIESTLEY, assistant lecturer in mathematics at the University of Manchester, has been appointed professor of mathematics and physics at the new University of Queensland.

PROFESSOR G. CASTELNUOVO, of the University of Rome, and Professor U. DINI, of the University of Pisa, have been elected to membership in the royal institute of Venice.

PROFESSOR M. ABRAHAM, of the technical school of Milan, has been elected a member of the royal institute of Lombardy.

DR. H. LEBESGUE, of the University of Poitiers, has been appointed master of mathematical conferences at the University of Paris.

PROFESSOR L. LECORNU, of the Ecole polytechnique, has been elected a member of the academy of sciences of Paris.

PROFESSOR A. EINSTEIN, of the technical school of Zürich, has been appointed professor of mathematical physics at the German University of Prague.

DR. L. BIEBERBACH has been appointed docent in mathematics at the University of Königsberg.

DR. R. NEUENDORFF has been appointed docent in mathematics at the University of Kiel.

DR. H. TIETZE, of the University of Vienna, has been appointed associate professor of mathematics at the German technical school at Brünn.

PROFESSOR G. B. HALSTED has been elected corresponding member of the academy of sciences of Bordeaux.

PROFESSOR JACQUES HADAMARD, of Paris, will deliver at Columbia University in the month of October, 1911, two courses of lectures on mathematics and mathematical physics. A course of weekly lectures open to the mathematical public is also planned. Further details will be announced later.

AT the University of California, associate professor G. C. EDWARDS has been promoted to a full professorship of mathematics.

AT Princeton University Dr. G. D. BIRKHOFF has been advanced to a full professorship of mathematics.

MR. J. B. SMITH, of the University of Virginia, has been appointed assistant professor of mathematics at Hampden Sydney College.

AT the University of Pennsylvania, Dr. H. B. SMITH has been appointed instructor in mathematics to fill the vacancy caused by the temporary absence of Professor H. B. EVANS.

PROFESSOR JULES TANNERY, of the Ecole normale, died November 11, at the age of 72 years.

PROFESSOR K. LASSWITZ, of Gotha, died October 17, at the age of 62 years.

PROFESSOR JULIUS PETERSEN, of the University of Copenhagen, died August 5, 1910, at the age of 71 years.

PROFESSOR TH. N. THIELE, former director of the observatory at Kopenhagen, died September 26, at the age of 71 years.

CATALOGUES of second-hand mathematical books: A. Hermann et Fils, 6 rue de la Sorbonne, Paris, catalogues in exact and natural sciences and in physics, 1910, about 160 entries in mathematics.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

AHRENS (W.). Mathematische Unterhaltungen und Spiele. 2te, vermehrte und verbesserte Auflage. In 2 Bänden. 1ster Band. Leipzig, Teubner, 1910. 8vo. 9+400 pp. M. 7.50.

BERNDT (F.). Grundzüge der Differential- und Integralrechnung. 4te Auflage. Leipzig, Weber, 1910. M. 3.00.

BIEDENKAPP (G.). Sophie Germain, ein weiblicher Denker. Jena, Tauscher, 1910.

BILLY (JACOBI DE). Doctrinae analyticae Inventum novum, Fermats Briefen an Billy entnommen. Herausgegeben und übersetzt von P. von Schaewen. Berlin, Salle, 1910. 143 pp. M. 3.00.

CALCULUS made easy. By F. R. S. London, Macmillan, 1910. 12mo. 188 pp. 2s.

CHARLIER (L.) und ZINNER (E.). Entwicklung des Potentials im Rotationsproblem. Stockholm, Almquist, 1910.

CLEBSCH (A.). Vorlesungen über Geometrie. Bearbeitet und herausgegeben von F. Lindemann. 2te, vermehrte Auflage. 1ter Band. 1ter Teil. 2te Leiferung. Leipzig, Teubner, 1910. 8vo. pp. 481–768. M. 9.00.

ENRIQUES (F.). Probleme der Wissenschaft. 2 Teile. 1ter Teil: Wirklichkeit und Logik. Übersetzt von J. Grelling. (Wissenschaft) und Hypothese. Band XI, 1.) Leipzig, Teubner, 1910. 10+258 pp. M. 4.00.

ENZYKLOPÄDIE der mathematischen Wissenschaften. 3ter Band: Geometrie. Redigiert von W. F. Meyer. 1ter Teil, 4tes Heft. Leipzig, Teubner, 1910. 8vo. pp. 481–770 M. 9.00.

FORTIN (C.). Théorie et applications élémentaires des vecteurs. Paris, Gigord, 1910. 16mo. 7+65 pp.

HENSEL (K.). Ernst Eduard Kummer und der grosse Fermatsche Satz. (Marburger akademische Reden, Nr. 23.) Marburg, Elwert, 1910. 8vo. 22 pp. M. 0.50.

KLEIN (F.) und SOMMERFELD (A.). Ueber die Theorie des Kreisels. 4tes Heft: Die technische Anwendungen der Kreiseltheorie. Für den Druck bearbeitet und ergänzt von F. Noether. Leipzig, Teubner, 1910. 8vo. 761–966 pp. Cloth. M. 9.00.

KOWALEWSKI (G.). Einführung in die analytische Geometrie. Leipzig, Veit, 1910. 8vo. 7+360 pp. Cloth. M. 11.00.

LINDEMANN (F.). See CLEBSCH (A.).

LORENZ (H.). Einführung in die Elemente der höheren Mathematik und Mechanik. München, Oldenbourg, 1910. 8vo. 5+176 pp. Cloth. M. 2.40.

LÜDERS (O.). Ueber orthogonale Invarianten der bizirkularen Kurven vierter Ordnung. (Diss.) Halle, 1910.

NERNST (W.) und SCHOENFLIES (A.). Kurzgefasstes Lehrbuch der Differential- und Integralrechnung mit besonderer Berücksichtigung der Chemie. 6te, vermehrte und verbesserte Auflage. München, Oldenbourg, 1910. 8vo. 12+442 pp. Cloth. M. 14.00.

D'OCAGNE (M.). Notions élémentaires sur la probabilité des erreurs. Paris, Gauthier-Villars, 1910. 26 pp.

PARFENTIEFF (N. N.). Études sur la théorie de la croissance des fonctions (en russe). Kasan, University, 1910.

PRASAD (G.). A textbook of integral calculus with numerous worked examples. London, Longmanns, 1910. 8vo. 5s.

SCHAEWEN (P. von). See BILLY (JACOBI DE).

SCHOENFLIES (A.). See NERNST (W.).

SOMMERFELD (A.). See KLEIN (F.).

WOLFF (G.). Über Kollineationen in der Ebene. (Diss.) Giessen, 1910.

WOLFF (H.). Formelsammlung zur analytischen Geometrie. Berlin, Maass und Plank, 1910. 8vo. 28 pp. M. 0. 70.

ZINNER (E.). See CHARLIER (L.).

II. ELEMENTARY MATHEMATICS.

AMIOT (A.). Trattato di geometria elementare. Nuova edizione, con aggiunte e modificazioni per cura di A. Socci. Settima impressione. Firenze, Le Monnier, 1910. 8vo. 8+402 pp. L. 2.50.

BECKER (E.). Logarithmisch-trigonometrisches Handbuch auf 5 Decimalen. 3te Ausgabe. Leipzig, Tauchnitz, 1910. 8vo. 16+104 pp. M. 1.20.

BERICHTE über den mathematischen Unterricht in Österreich. 2tes Heft: von T. Konradt, M. Dolinski, und M. Adamička. Wien, Hölder, 1910. 8vo. 52 pp. M. 1.20.

BERTRAND (G.). Trattato di algebra elementare. Nuova edizione, con aggiunte e modificazioni per cura di A. Socci. Quattordicesima impressione. Firenze, Le Monnier, 1911. 16mo. 7+531 pp. L. 3.00.

BOREL (E.) et MONTEL (P.). Algèbre. (Cours de mathématiques Émile Borel.) Paris, Colin, 1910. 16mo. 337 pp. Fr. 3.00.

BOURLET (C.). Éléments d'algèbre, contenant 561 exercises et problèmes. 7e édition, revue. Paris, Hachette, 1910. 16mo. 295 pp. Fr. 2.00.

——. Éléments d'algèbre. Corrigés des 561 exercices avec la collaboration de J. Hulot. 3e édition, revue. Paris, Hachette, 1910. 16mo. 355 pp. Fr. 3.00.

——. Éléments de géométrie, contenant 762 exercices. 2e édition, revue. Paris, Hachette, 1910. 16mo. 6+383 pp. Fr. 2.50.

BRUCE (W. H.) and CODY (C. C.). Elements of plane geometry. Dallas, Tex., Southern Publishing Co., 1910. 12mo. 263 pp. $0.75.

CATANIA (S.). Corso di algebra elementare. Volume II. Seconda edizione, corretta e accresciuta. Catania, Giannotta, 1910. 16mo. 278 pp. L. 2.00.

——. Trattato di aritmetica ed algebra. Terza edizione, migliorata e corretta. Catania, Giannotta, 1910. 16mo. 16+576 pp. L. 4.50.

CAVAZZONI (L.) E CERCIGNANI (E.). Libro di geometria del piano. Milano, Albrighi, 1911. 8vo. 300 pp. L. 2.80.

CERCIGNANI (E.). See CAVAZZONI (L.).

CODY (C. C.). See BRUCE (W. H.).

COURS d'algèbre élémentaire conforme aux derniers programmes de l'enseignement secondaire. Par F. G. M. 6e édition. Paris, Poussielgue, 1911. 8vo. 8+568 pp.

CRANTZ (P.). Lehrbuch der Mathematik für höhere Mädchenschulen and Lyzeen. 1ter Teil. 3te Auflage. Leipzig, Teubner, 1910. 8vo. 6+177 pp. Cloth. M. 2.40.

DAUCHY (F.). See PHILIPPE (P.).

FRANCHIS (M. DE). Elementi di trigonometria rettilinea. Milano, Sandron, 1910. 8vo. 66 pp. L. 1.00.

HUE (T.) et VAGNIER (M.). Géométrie, enseignement général. Édition refondue, conforme au programme de 1909. (Bibliothèque publiée sous la direction de F. Martel.) Paris, Delagrave, 1910. 18mo. 557 pp. Fr. 3.90.

JENSON (O.). See WALTHER (F.).

KAMBLY-ROEDER. Planimetrie nebst Elementen der Trigonometrie und Stereometrie. Neu bearbeitet von A. Thaer. Ausgabe C. Breslau, Hirt, 1910. M. 2.75.

LADNER (J. W.). The invicta table book. Graphic representations of the multiplication tables and of the commoner weights and measures. London, Philip, 1910. 18mo. 16 pp. Sewed. 2s.

LANNER, (A.). Neuere Darstellungen der Grundprobleme der reinen Mathematik im Bereiche der Mittelschule. Berlin, Salle, 1910. M. 3.00.

LAYING (A. E.). A general textbook of elementary algebra. Exercises, book 2. With answers. London, Blackie, 1910. 8vo. 2s.

LEPOIVRE (G.) et POIRSON (A.). Cours de géométrie théorique et pratique. 2e édition, revue et augmentée. Lille, Janny, 1910. 8vo. 6+213 pp. Fr. 2.50.

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POIRSON (A.). See LEPOIVRE (G.).

PONZER (E. W.). Graphs for reference. Stanford University, Cal., Ponzer, 1910. 12mo. 12pp., 82 figures. Paper. $0.10.

ROPERT (J.). Cours d'algèbre et de géométrie. Notes de géométrie descriptive. Angers, Lenormand, 1910. 4to. 407 pp.

SCHWAB (K.) und LESSER (O.). Mathematisches Unterrichtswerk zum Gebrauche an den höheren Lehranstalten für die weibliche Jugend. 2ter Band: Geometrie, bearbeitet von M. Linnich. 1ter Teil. Leipzig, Freytag, 1910. 8vo. 212 pp. Cloth. M. 2.80.

——. Mathematisches Unterrichtswerk. Arithmetik und Algebra. Bearbeitet von M. Linnich. Leipzig, Freytag, 1910. 8vo. 177 pp. Cloth. M. 2.50.

SICKENBERGER (A.). Leitfaden der elementaren Mathematik. 2ter Teil: Planimetrie. 7te Auflage, bearbeitet von A. Schmid. Nürnberg, Koch, 1911. 8vo. 4+126 pp. M. 1.65.

——. Übungsbuch zur Algebra. Zwei Abteilungen. Bearbeitet von A. Schmid. Nürnberg, Koch, 1911. 8vo. M. 3.35.

THOMAS (D.). Practical geometry for junior forms. With answers. London, Simpkin, 1910. 16mo. 1s. 4d.

VAGNIER (M.). See HUE (T.).

WALTHER (F.). Mathematischer Lehr- und Übungsgang für Lehrerbildungsanstalten. Drei Teile. Leipzig, Brandstetter, 1910. 8vo. M. 7.60.

WALTHER (F.). und JENSON (O.). Mathematischer Lehr- und Übungsgang für höhere Mädchenschulen. Zwei Abteilungen. Leipzig, Brandstetter, 1910. 8vo. 8+199+135 pp. M. 4.60.

ZIMMERMANN (L.). Mathematische Formelsammlung. Zur Vorbereitung für das Einjährig-Freiwilligen-Examen. Essen, Baedeker, 1910 8vo. 4+55 pp. M. 1.50

III. APPLIED MATHEMATICS.

BARCHANEK. Darstellende Geometrie und Raumlehre. Bearbeitet von E. Ludwig. 3te, nach den neuen Lehrplänen umgearbeitete Auflage. Wien, Tempsky, 1910. 8vo. 247 pp. Cloth. M. 4.00.

BOERNER (F.). Statische Tabellen, Belastungsangaben und Formeln zur Aufstellung von Berechnungen für Baukonstruktionen. 3te, nach den neuesten Bestimmungen bearbeitete Auflage. Berlin, Ernst, 1910. M. 4.20.

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ENZYKLOPÄDIE der mathematischen Wissenschaften. Band VI, 1 B: Geodäsie und Geophysik. Redigiert von Ph. Furtwängler und E. Wiechert. 2tes Heft. Leipzig, Teubner, 1910. 8vo, pp. 85–177. M. 3.00.

FLEMING (J. A.). The principles of electric wave telegraphy and telephony. 2nd edition, revised and extended. London, Longmans, 1910. 8vo. 926 pp. 28s.

FÖPPL (A.). Vorlesungen über technische Mechanik. In 6 Bänden. 6ter Band: Die wichtigsten Lehren der höheren Dynamik. Leipzig, Teubner, 1910. 8vo. 12+490 pp. M. 12.00.

FREUND (A.). Technische Elementarmechanik fester Körper. 2te Auflage. Leipzig, Brandstetter, 1910.

HARTMANN (F. M.). Elementary mechanics for engineering students. New York, Wiley, 1910. 12mo. 171 pp. Cloth. $1.25.

HERTZ (H.). Gesammelte Werke. 3ter Band: Die Prinzipien der Mechanik in neuem Zusammenhange dargestellt. Herausgegeben von P. Lenard. 2te Auflage. Leipzig, Barth, 1910 32+312 pp. M. 13.50.

HOHENNER (H.). Geodäsie. Eine Anleitung zu geodätischen Messungen. (Sammlung von Lehr- und Handbüchern, herausgegeben von. F Doflein und K. T. Fischer.) Leipzig, Teubner, 1910. 8vo. 12+347 pp. Cloth. M. 12.00.

HOLTON (E. E.). Shop mathematics. (Haytol series of text-books for industrial education, edited by F. E. Mathewson.) Springfield, Mass., Taylor-Holden Co., 1910. 12mo. 11+211 pp. $1.25.

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HORT (H.). Der Entropiesatz. Berlin, Springer, 1910. M. 1.00.

HOSMER (G. L.). Textbook on practical astronomy. New York, Wiley, 1910. 8vo. 9+205 pp. Cloth. $2.00.

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JORDAN (W.). Handbuch der Vermessungskunde. 1ter Band: Ausgleichungsrechnung nach der Methode der kleinsten Quadraten. 6te erweiterte Auflage, bearbeitet von O. Eggert. Stuttgart, Metzler, 1910. 11+596 pp.

KARAPETOFF (V.). Experimental electrical engineering and manual for electrical testing for engineers and for students in engineering laboratories. Volume I. 2nd edition, corrected. New York, Wiley, 1910. 470 pp. Cloth. $3.50.

KENT (W.). The mechanical engineer's pocket-book. A reference book of rules, tables, data, and formulas, for the use of engineers, mechanics, and students. 8th edition. rewritten and enlarged. New York, Wiley, 1910. 16mo. 40+1461 pp. Morocco. $5.00.

LANGE (M.). Das Schachspiel und seine strategischen Prinzipien. (Aus Natur und Geisteswelt. Band 281.) Leipzig, Teubner, 1910. 8vo. 107 pp. M. 1.25.

LEROY (C. F. A.). Traité de stéréotomie, comprenant les applications de la géométrie descriptive a la théorie des ombres, etc., 14e édition, revue et annotée par M. E. Martelet. Supplément par J. de la Gournerie. 2 tomes. Paris, Gauthier-Villars, 1910. 16+428+76 pp. Fr. 26.00.

LORENTZ (H. A.). Sichtbare und unsichtbare Bewegungen. Ubersetzt von G. Siebert. 2te Auflage. Braunschweig, Vieweg, 1910. M. 4.00.

MAXIM (J. L.). Experimental applied mechanics for technical students. London, 1910. 12mo. 176 pp. Cloth. $0.60.

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PIZZETTI (P.). Tabelle grafiche per la risoluzione approssimata di una equazione di Gauss che si incontra nel calcolo delle orbite. Pisa, Nistri, 1910. 8vo. 6 pp.

ROBERTS (S. S.). Track formulae and tables. New York, Wiley, 1910. 16mo. 12+514 pp. Morocco. $3.00.

SCHIFFNER (F.). Leitfaden für den Unterricht in der darstellenden Geometrie. 2 Teile. Wien, Deuticke, 1910. 8vo. Cloth. M. 3.40.

——. Geometrisches Zeichnen. Wien, Deuticke, 1910. 8vo. 4+75 pp. M. 1.20.

SEE (T. J. J.). Researches on the evolution of the stellar systems. Volume 2: The capture theory of cosmical evolution. London, Wesley, 1910. 4to. 7+734 pp. 42s.

SILVESTRI (E.). Esercitazioni di idraulica. Torino, Pasta, 1910. 8vo. 157 pp. L. 5.00.

SMITH (L. C.). See JOHNSON (J. B.).

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TIEDT (G.). Messungen der Abhängigkeit für Elastizitätsgrenze und Elastizitätszahl von verschiedenen Einflüssen. (Diss.) Marburg, 1910.

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WOLFF (H.). Nivellieren. Formulare und Berechnungen. Berlin, Maas und Plank, 1910. 8vo. 5+69 pp. M. 2.00.